УДК 539.25

SOUND WAVE PROPAGATION ALONG THE WEDGE EDGE

Kh. B. Tolipov

There is given a theoretical research of Rayleigh wave's propagation in an elastic wedge. The dependencies of propagation coefficient got are in good correlation with well–known experimental data. There is given a theoretical research of Rayleigh wave's transformation of surface waves in an elastic wedge. The dependencies of transformation coefficient got are in good correlation with well–known experimental data.

Formulation. Let's assume that an elastic media characterized by Lame constants λ, μ and density ρ_0 fills infinite wedge shaped domain $\Omega\,(-\theta_0,\ \theta_0) = (r \geq 0,\ -\theta_0 \leq \theta \leq \theta_0,\ z > 0)$, where z – is directed along the wedge edge. Differential equation of particles' movement in absence of volume forces in the form of a vector could be represented in the form:

$$(\lambda + \mu) graddivU + \mu\Delta U = \rho_0 U''. \tag{1}$$

Expressing vector potential in terms of 2 scalar functions ψ_1 and ψ_2, let's express a displacement vector in the form:

$$\mathrm{U} = \mathrm{grad}\,\varphi + rot\,\psi_1 k + rotrot\,\psi_2 k \tag{2}$$

where φ – scalar potential; k – unit vector along the z-axis.

In this case, solution of a vector wave equation is separated for each function ψ_1 and ψ_2:

$$\Delta\varphi + K_l\varphi = 0, \qquad \Delta\psi_1 + K_t\psi_1 = 0, \qquad j = 1,2, \tag{3}$$

where

$$\Delta = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}, \qquad K_h = \frac{\omega}{C_h},$$

here C_l, (C_t) – propagation speed of longitudinal and transverse waves, ω – angular frequency.

No density condition $\sigma_{\theta\rho} = \sigma_{\theta z} = \sigma_{\theta\theta} = 0$ is observed on free wedge boundaries $\theta = \pm\theta_0$. Let's also assume, that particles' movement starts from a state at rest.

It should be noted that to formulate a problem to a motion equation correctly, there should be boundary conditions, radiation conditions as well as a condition, regulating the behavior of the unknown functions in the vicinity of singular points, close to the edge.

This condition is equivalent to the requirement to observe the law of conservation of energy, and can be given in the following form:

$$U = 0(\rho^p), \qquad \rho \to 0$$

2. Solution of the problem. Let's study the solutions, corresponding to the harmonic waves running towards axis z in the form:

$$\varphi = (A_0 ch\,\nu_1\theta + C_0 sh\,\nu_1\theta) H_{\nu 1}^{(1)}(\rho\alpha)\exp i(pz - \omega t),$$
$$\psi_1 = (A_1 ch\,\nu_2\theta + C_1 sh\,\nu_2\theta) H_{\nu 2}^{(1)}(\rho\beta)\exp i(pz - \omega t), \tag{4}$$

where

$$\alpha = \sqrt{p^2 - K_l^2},\ \beta = \sqrt{p^2 - K_t^2}, \quad p > 0,$$

here p – wave number, ν_1 and ν_2 – angular wave numbers, J_{ν_1} and J_{ν_2} – Bessel functions.

Further, factor $i(z - \omega t)$ is dropped.

Using (2) and Hook's Law for an isotrope body, it better to give tensor tensions' components in terms of wave potentials:

$$\frac{\sigma_{\theta\rho}}{\mu} = \frac{2}{\rho}\frac{\partial^2\varphi}{\partial\rho\partial\theta} - \frac{2}{\rho^2}\frac{\partial\varphi}{\partial\theta} + \frac{1}{\rho}\frac{\partial\psi_1}{\partial\rho} - \frac{\partial^2\psi_1}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2\psi_1}{\partial\theta^2} + \frac{2}{\rho}\frac{\partial^3\psi_2}{\partial\rho\partial\theta\partial z} - \frac{2}{\rho^2}\frac{\partial^2\psi_2}{\partial\theta\,\partial z},$$

$$\frac{\sigma_{\theta z}}{\mu}=\frac{2}{\rho}\frac{\partial^2\varphi}{\partial\theta\partial z}-\frac{\partial^2\psi_1}{\partial\rho\partial z}+\frac{1}{\rho}\frac{\partial^3\psi_2}{\partial z^2\partial\theta}-\frac{1}{\rho^2}\frac{\partial^2\psi_2}{\partial\rho\partial\theta}-\frac{1}{\rho^2}\frac{\partial^3\psi_2}{\partial\theta^3}-\frac{1}{\rho}\frac{\partial^3\psi_2}{\partial\theta\partial\rho^2},$$

$$\frac{\sigma_{\theta\theta}}{2\mu}=\frac{\partial^2\varphi}{\partial z^2}-\frac{K_t^2}{2}\varphi-\frac{\partial^2\varphi}{\partial\rho^2}-\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial\psi_1}{\partial\theta}\right)+\frac{1}{\rho}\frac{\partial^2\psi_2}{\partial\rho\partial z}+\frac{1}{\rho^2}\frac{\partial^3\psi_2}{\partial\theta^2\partial z}$$

Substituting expressions (4) with six constants A_0, A_1, A_2, C_0, C_1 and C_2 into (5) we obtain an algebra system of six uniform equations to bind A_0, A_1, A_2, C_0, C_1 and C_2.

This system has solutions corresponding to symmetric and antisymmetric with respect to plane $\theta =$ waves.

A condition of existence of nontrivial solutions of this equation system is that determinant, which represented in the form of a product $\Delta_s \cdot \Delta_{as}$, where Δ_{as} corresponds to antisymmetric, and Δ_s – to symm ric waves, is equal to zero. This equation has a solution at two independent conditions $\Delta_s = 0$ and $\Delta_{as} =$ which determine two different kinds of surface waves. Let's consider the first kind of waves.

3. Antisymmetric case. Let's assume, that there are no disturbance oscillations in the vicinity wave' propagation along ρ axis. Then, neglecting the derivatives of ρ coordinate, the equation for t case will have the following form:

$$\frac{(p^2+\beta^2)^2}{4\alpha\beta p^2}-\left[\frac{th\nu_2\theta}{th\nu_1\theta}\right]^b=0,$$

$$\nu_1=\sqrt{p^2-K_t^2}\cdot\rho,\ \nu_2=\sqrt{p^2-K_t^2}\cdot\rho$$

It's not difficult to see, that at big values of a wedge angle or a distance to its edge equation transfers into the known Rayleigh equation.

For a compete solution in the boundary angular vicinity, it is necessary to include the condition the edge:

$$U_\rho=\frac{\partial\varphi}{\partial\rho}+\frac{\partial^2\psi_2}{\partial\rho\partial z}=const=A,\ \rho\to\alpha,$$

where A – is constant.

Using the result of [5], it's possible to determine the value of displacements' amplitudes.

We'll use Fourier transformation on ρ coordinate for equation (3) and (5).

Then, the solution of the wave equation, simultaneously satisfying the solution on the edge and boun conditions, can be given in the form:

$$\tilde\varphi(k_r)=\frac{AJ_{\nu_1}(K_\ell\rho)}{BJ^{/}_{\nu_1}(K_\ell\rho)+iCJ^{/}_{\nu_2}(K_t\rho)p},\quad \tilde\psi(k_r)=\frac{AJ_{\nu_2}(K_t\rho)}{BJ^{/}_{\nu_1}(K_t\rho)+iCJ^{/}_{\nu_2}(K_t\rho)p},$$

where $J_\nu(x)$ – Bessel function of the first kind, A, B, C – constants, B and C are interconnecte boundary conditions on the wedge surface, and primes denote derivatives of ρ coordinate.

Transferring inverse Fourier transformation into a complex plane, we obtain solutions, correspondir residues in p_n poles of an integrand expression.

$$\varphi=\sum_{n=0}^{\infty}A_n e^{ip_n z},\qquad \psi=\sum_{n=0}^{\infty}B_n e^{ip_n z},$$

where

$$A_n=\frac{J_{\nu_1}(K_\ell\rho)}{\left(\frac{\partial^2 J_{\nu_2}}{\partial\rho\partial p}\right)_{p=p_n}},\qquad B_n=\frac{J_{\nu_2}(K_t\rho)}{\left(\frac{\partial^2 J_{\nu_2}}{\partial\rho\partial p}\right)_{p=p_n}}.$$

This solution, called waveguide solution, forms a discrete field spectrum. Values of $K_t\rho$ roots the denominator in equation (6) could be called «width» of the waveguide. Values of phase velocit depend on this parameter and are found from equation (8). Fig. 1 shows distributions of amplituc normal waves of the second and sixth order, correspondingly, with wedge angle $\theta = 15°$.

Fig. 2 gives angular dependencies of antisymmetric wave's velocity of the second and sixth order.

Discussion of result as the analysis shows, disturbances with a very low velocity can propagate the wedge edge. With an increase of a wedge angle, the velocity of these disturbances increases, ar

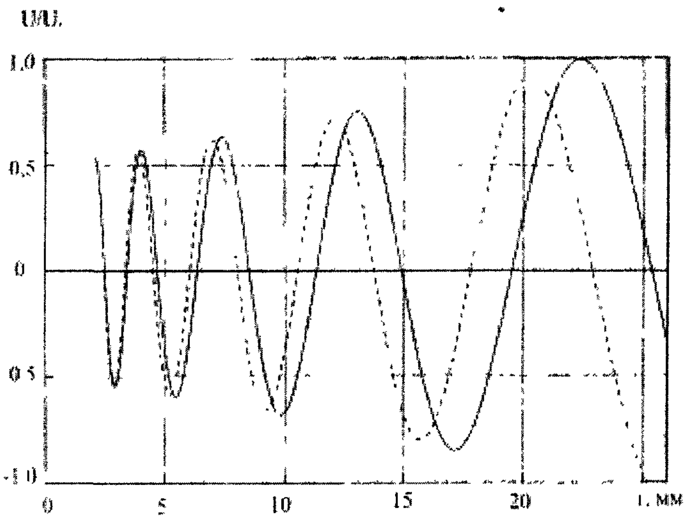

Fig. 1. Amplitudes of normal waves of the second and sixth order, correspondingly, with wedge angle θ = 15°

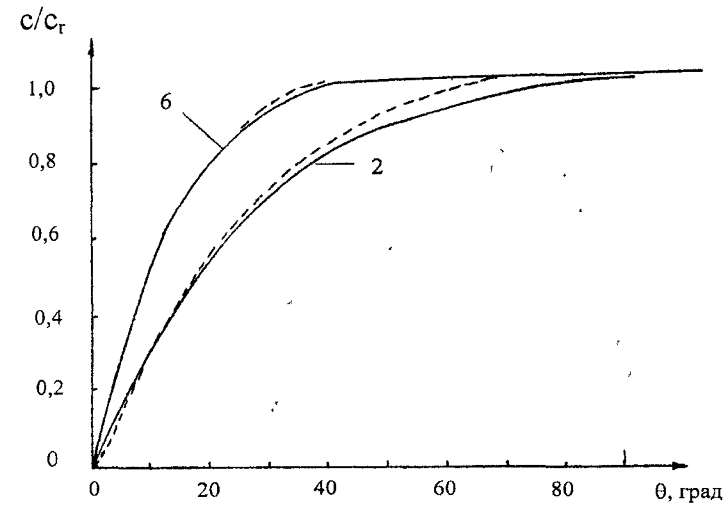

Fig. 2. Angular dependencies of antisymmetric wave's velocity

equal to velocity of Rayleigh wave at angles $2\theta \geq 90°$. An increase of velocity of a wave with a wedge angle speaks about an increase of media's toughness, corresponding to a big local wedge thickness.

Propagation of an antisymmetric wave along the edge takes place in a narrow wave channel. When disturbance diverge from the route of wave propagation into the media there happens a refraction of an antisymmetric wave because of a velocity decrease. As a result, there's observed a turn of rays towards the top of the wedge and, a return of waves to the wave channel. That's why this wave is acoustically stable in a narrow wave channel.

Propagation of a symmetric wave is of another character. Disturbance velocity of wedge media's particles, when approaching the edge increases monotonically up to the velocity of volume waves (see [6]). At a small divergence form the wave's propagation route the velocity of the wave decreases, and the path of disturbances is curving into the media. Thus, this wave is acoustically instable in a wave channel.

References

1. Kane J., Spence J. Rayleigh wave transmission in elastic wedge // Geophysics. – 1963. – V. 28, № 5, pt. 1. – P. 715–723.

2. Budaev B.V., Bogy D.B. Rayleign wave scattering a wedge // Wave Motion, Nov 1996.

3. Budaev B.V., Bogy D.B. Scattering of Rayleign and Stoneley waves by two adhering elastic wedges // Wave Motion, Apr 2001.

4. Budaev B.V., Bogy D.B. Errata to the paper Rayleign wave scattering by a wedge // Wave Motion, Mar 2002.

5. Viktorov I.A. Physical Principles of the Applications of Rayleigh and Lamb Ultrasonic Waves in Technology. – Moscow: Nauka, 1966.

6. Tolipov Kh.B. A dynamical problem of the theory of elasticity for angular domains with homogeneous boundary conditions // J. Appl. Maths Mechs. – Vol. 57, №.5. – P. 879–885, 1993.

Поступила в редакцию 4 апреля 2003 г